DOCUMENT RESUME

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These materials were written with the aim of reflecting the thinking of Cambridge Conference on School Mathematics (CCSM) regarding the goals and objectives for school mathematics. Presented are plans for teaching 23 probability lessons in the elementary grades at Hancock School, Lexington, Massachusetts. The discovery approach was utilized by the teacher to involve students in the classroom discussions. Tossing a coin, rolling a die, tossing a thumbtack, and graphing are mediums used in the classroom by students before responding to questions presented by the teacher. The lesson plan for each day is listed. Transcripts of teacher and student statements are included for several lessons. [Not available in hard copy due to marginal legibility of original document]. (RP)

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CCSM
Feasibility Study No. 41

Probability Lessons at Hancock School, Lexington

Taught by Miss Lyn McLane, Co-ordinator of grades 4-6 Mathematics, Lexington Schools.

Class Teacher: Miss Lee Kubasciewicz

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SE007 201

Plans for 1st Lesson on November 29, 1966

1. Discuss what probability is.
2. Tossing a coin
   a. Getting a head after 3 tails
   b. Idea that prob of head is the same for each toss but over long range

      # heads ≠ # tails = 1
3. Rolling a die
4. Tossing a thumbtack - point up ( ) or down ().
   a. How could we find out what the probability of getting a point up is?
   b. Do we need to worry about how we toss it?
   c. How about slow pushes off table.
   d. Set up plan of attack.
5. Graphing
   a. St line through origin on graph paper
   b. What is equation?
   c. How could we make a steeper line?
   d. How could we make a less steep line?
   e. □×□ =△ Draws graph.

Probability Tape 1st Lesson November 29, 1966

McLane - Hancock 6AP

T: Try something we haven't done before. What does word probability mean? What is probability used for?

S_1: Used in playing games.

S_2: Most likely.

S_3: Comes from "probable"

Missed recording whole section of tape after break in tape.

Flipping '64 nickel, class did not like way in which Bob flipped the nickel so Betsy tried it.

T: What did you end up with?

S. 6-4

T. Betsy 6H 4T

On board	Bob	4H
	Betsy	6H

T: What is probability of getting heads?

Charlie - On each flip you still don't know which will come up.

David - 60% heads one time, 40% heads the other - so they average out to 50-50.

T: How many throws all together?

S: 20

T: How many heads?

David: Ten.

Paul: Coin is loaded.

S: Betsy did you flip nickel with heads on top always?

Betsy: Yes.

Bob: However coin fell is way I flipped it.

Susan: (Describing some experiment - tossing object)

Kathy: I saw that too but I didn't know what it was.

Bob: I think everything should be tried again exactly as before. To see if there is some rule.

T: How many times do you have to flip it to see whether it just happened or whether there is some law?

S: Do thing twice over again ...

T: While they are flipping I'm going to draw graph.

S: Another '64 nickel - they are just the same
(Murmurs to the contrary from the class.)

Bob: T T T

T: How many think he is going to end up with 4 H again? Only three of you?

Bob: H

Class: Yeah.

Bob: H

Bob: H

Bob: (Tells results - ends up with 3H)

Betsy flips

(If coin fell on floor it wasn't counted - she had to catch it.)

Ends up with 7H

On Board

Bob	4H	3H
Betsy	6H	7H

S: Chance is still 50-50

Bob: I don't know that there is an actual rule of the chances but I do know that our two ways are exact opposites.

Betsy: I wonder, if I flipped first, I wonder how it would come out.

Susie: The second time changed from the first time so there isn't any law.

Charlie: I still say it's a 50-50 chance. You cannot judge what it is going to come out. Sometimes it comes out what you say it is going to come out but I mean it is just coincidental, but I don't think you are going to get anything else but heads or tails.

S: First time Bob had 4H 6T, Betsy had 6H 4T, 2nd time Bob had 3H 7T, Betsy had 7H 3T - There still is a 50-50 chance.

Bob: We haven't established any rule 40-60 chance or 50-50 but we have established that my way is exactly the opposite.

Betsy: If we flipped it differently - if I flipped it first it might turn out that I would get lower heads.

T: Would you like to try it?

T: (Betsy flips first - flipping the same way she did.)
Ended up with 5H.
Child kept track of order of tosses such as H H T H ... etc. on board.

Bob flipped.
Child kept track under Betsy's T H ... etc.

Ended up with 3H

On Board:

Bob	4H	3H	3H
Betsy	6H	7H	5H

T: Now what happened?

David: He just got 7T 3H. She got 5T 5H.
If you did it 100 times ...

T: Bob, what does that do to your theory?

Bob: Mine is consistent. The way I do it, it is always consistent. Well it is not always but it is close.

T: How close is close?

Bob: ?

Kathy: You have to flip infinity times to find out whether it is a 50-50 chance.

David: I say you have a 50% chance before you throw it. It doesn't make any difference what happens. You still have a 50% chance that it will come up heads or tails. It might have more heads for example but you still have 50% chance before you throw it.

T: Is that assuming the coin is not weighted in any way?

David: Yes.

Aliscn: Bob said that he thinks the numbers are consistent but you can't tell that because you only have three numbers. You have to try about 120 (?) times to see if it really was.

Helena: (too soft to hear)

T: I've got something here for next time I come. What would happen if we flipped this thing - a thumbtack.

S: It would land pt. up

S: It would land sideways.

S: 40-60

T: You think 40% of the time it will land up?

S: Yes.

Bob: Discussion of how thumbtack lands - way in which it first hits floor.

T: Charlie, I'd like to know what you think the probability is that this thing will land pt. up.

Charlie: ... but I think it is still a 50-50 chance. Even if it wasn't a 50-50 chance you can not tell .... even it lands 100 times or 1000 times it might start landing differently.

Bob: (Discussion of aerodynamics of thumbtack - like man with parachute shot out of cannon)

S: Take point of thumbtack - weigh head, weigh point the heavier one will come down first.

David: I think there is more chance of the point being up on its side than of the point being down.

T: I think we can probably not even worry about the point being down.

(Discussion of parachute)

T: What would be probability of having nail land point up?

Jeff: 10-1 odds it won't.

S: 99-1

S: More chance of it landing on its side because the head isn't very big.

S: 9,999 - 1

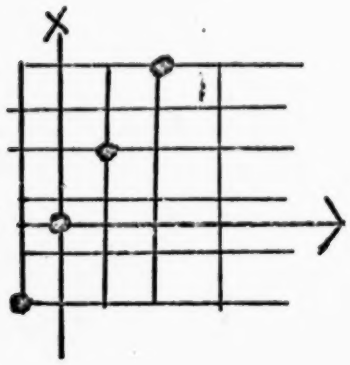
Bob: (Looking at nail at first pt. of contact)

David: 999,000 - 1. that nail will land on side.

S: (One end of nail will come down first - depends on how you throw it up)

T: Next time - we'll have thumbtacks and do something like we did here except all of you will be able to flip.

T. Find open sentence for



How to make it steeper?

(Some children had figured it out already)

Plans for 2nd Lesson on December 8, 1966

1. Discuss most desirable method of throwing thumbtack so as to give consistent results for whole class and to avoid flying thumbtacks. Perhaps push thumbtack off desk top.

2. Divide class into teams of 4 students each. Give each student mimeographed sheet for recording results of individual tosses as well as sheet for recording data for his team.

   Each student will:

   a. Toss Thumbtack 50 times.
   b. Find range r.
   c. Make frequency table.
   d. Make frequency graph.
   e. Find value of No. of Ups which gives peak N.

   Each team will:

   a. Reorganize data.
   b. Find range.
   c. Make frequency table.
   d. Make frequency graph.
   e. Find value of No. of Ups which gives peak N.

3. Record results of N and r for each student on table on board. Find average N and average r. (T=5)

4. Record results of N and r for each team.
   Find average N and average r. (T=20)

5. Discuss data and what would happen if we let T=45 or T=80?

6. Graphing - idea of slope of line - if there is time.

Tape of 2nd Lesson — December 8, 1966

McLane - Hancock 6AP

Discussion of flipping of coin

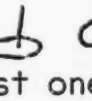
Charlie firm on idea of coincidence.
Tried to discuss what would happen if we flipped 10 times.
100 times.
1000 times.

Always 50% chance of heads before you flip.
[Maybe confusion over two possible outcomes and p=1/2]

David: You could flip it 1 million times and you still won't prove anything except that it is coincidence.

Thumbtack T: I will want everyone to toss thumbtack 50 times.

T: How can we be sure everyone is tossing it same way.

S: Gadget to flip it.
Possible outcomes 
Point down (last one) was discarded.

T: Everyone flip thumbtack for two minutes and try to find best way of tossing it so that everyone will be tossing it the same way and so that no one will get injured by flying tacks.

Bedlam as students start tossing tacks..
Most students started keeping track of outcomes.

T: We have to agree on one way of doing it.

T: Make list of different ways

1) Toss from palm of hand
2) Hold pt. between thumb and forefinger
3) Flip from thumbnail starting with pt. up
4) Pt. sideways between thumb and forefinger
5) Same as No. 2 but hard on top.
6) Use big paper clip tack in hole - use paper clip as lever.
7) Push off desk
8) Ruler - balance tack on top of ruler
9) Ruler as lever

Speeches on best way.

S: Some desks are higher than others so No. 7 is not very good.

Vote:			
	No. 1	9	Tossing it up 1-1/2' so it lands on desk.
	No. 2	1	
	No. 3	9	
	No. 4	0	
	No. 5	0	
	No. 6	0	
	No. 7	0	
	No. 8	5	

Not finished yet

Plans for 3rd Lesson on December 13, 1966

1. Discuss graph of last time. $\square \times 2 = \triangle$

2. Pass back sheets containing results of their fifty tosses of the thumbtack.

3. Discuss range, N, and graph. What do they tell us about the data? What might we expect average values for class to be? What might the graph look like if we allow more tosses in each set?

4. Record r, N for each student. Find averages value for each and record on table for T = 5.

5. Pass out sheets for T = 20 to each team. Each team should fill out table, find range, make frequency table, make frequency graph.

6. If time, record results of teams and find averages.

December 13, 1966

Thumbtack T = 5

Name	r	N
Alison Bernstein	4	3,4
Helena Donovan	3	4
Alec Rawis	1	3
Teddy Schatzki	4	1,2
Nancy Asarkof	2	3
Betsy Rising	4	3
Susan Guthe	4	3
Roger Tucker	3	5
Brian Burrell	3	5
Stephen Blackmer	2	4
Jef Fowle	3	4
Bob Brehm	3	3
Cathy Mann	3	4
Jody Hogt	3	4
Sheila Quinlan	3	2
Paul Cobb	2	3,4
Kim Winnard	4	1
David Mack	2	4
Charlie Alberts	4	5
David Relman	4	2
Susan Butler	3	5
Karen Agresti	2	2
George Bryant	4	2
Total number	70	76.5

R
Total average = 3
Total number = 70

N
Total average = 3.3
Total number = 76.5

December 13, 1966 - Third Lesson

Discussion as to what graph will look like.
Agree on straight line along ☐ axis.

Handback papers on thumbtacks.

T: How do we describe range?

T: How about peak? Could all of you find it?

T: What would we get if found the average peak in whole class?

S: It probably wouldn't be equal.

T: What do you mean by equal?

S: Not 50-50

T: What do you think average would be?

Rob: 4-1/2 - 5

T: What would range and peak be if we regrouped data into sets of 20?

S: Average number of ups 15 - 17.

T: What would frequency graphs look like? - More bumps, smoother?

Hand out sheets for regrouping into sets of 20.
Class organized into teams of 4 students.
Teams work on filling in sheet - see sheet.
Some had difficulty with frequency table.

T: May I have your attention please.
Alison is collecting values for N and r from last week.

T: Would someone collect all the papers please, Betsy.

T: Next time I want us to take a look at the two different graphs we have. What do they tell us?

Alison has ranges and peaks - we will find averages next time.
We will also find averages for ranges and peak for T = 20.

T: Could we do the same thing with a coin?

David: I think we can predict there is more of a coincidence . We could predict more with a thumbtack than with a coin.

T: You mean you can be more sure with a thumbtack than with a coin.

Name _____

Date _____

1. Use the data you have just collected individually to fill in the table below for your team. Write the names of your team members at the top of the columns indicated.

	No. of Ups for	No. of Ups for	No. of Ups for	No. of Ups for		Totals
Set 1						
Set 2						
Set 3						
Set 4						
Set 5						
Set 6						
Set 7						
Set 8						
Set 9						
Set 10						
					Total	

No. of Ups □	No. of Sets △
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	

2. Find the range. r = _____.

3. Fill in the table to the right to show how many sets had 0 ups, 1 ups etc.

   Use graph paper to make a graph of the pairs of numbers in the table.

4. Find the peak of the graph. What is the □ number which goes with the peak?

   N = _____.

Probability Plans for 4th Lesson on December 22, 1966

McLane - Hancock 6AP

1. Have one team find average N (no. of ups) and average r (range) for T = 5.

2. Make table on board to get values of N and r from each team. T = 20

Team	r	N

3. Find average r and average N for T = 20.

4. Tape graphs of each team on board so students can see results of the other teams.

Fourth Session (Lyn McLane - December 22, 1967) Hancock 6AP

T: The first time we tossed we organized into sets of 5.
Second time we organized into sets of 20.
I would like to see what happens if we look at the graphs and data for sets of 5 and 20.
Next time we will do sets of 45.
Find averages for range and peak.
I would like one group to work an average for the whole class.
Each team find the best graph.
I would like one graph from each team to be put on the board.
(Graph for T = 20)

	r	N
Betsy's Team	3	8
Helena	4	
Cathy		
David R.		

[couldn't hear numbers]

T: I'd like to have the rest of you find the average r and average N.

Disagreement on averages

Tiny Tim in the tape recorder.

T: Alec would you sit down piease.

T: There seems to be some dispute - average r.

Jody: 6-5/6

T: Can we find a decimal name for 5/6?

Cathy: 6.8 r.2

T: How would we write this as a fraction?

S: 6.8-1/3

T. What is decimal name for 1/3?

S. .333 . . .

T: So we could write 6.8-1/3 as 6.8333 ...
How many think these are all different names for the same number?

6-5/6, 6.8r.2, 6.8-2/6, 6.8-1/3, 6.8333 ...

T: 2-1/2 of you.

Discussion as to which ones are equal.

Some think 6-5/6 doesn't belong.

S: I can prove it belongs there.

$$
\begin{array}{r}
.833 \\
6\overline{)5.000} \\
48 \\
\hline
20 \\
18 \\
\hline
2
\end{array}
$$

Cathy: 6-5/6 = 6.833 ... = 6.8-1/3

T: Everyone had different answers and we want to show that they just look different.

Cathy: But they aren't different.

T:
$$6\overline{)41.00}\quad 6.83$$
$$\begin{array}{r} 36 \\ \hline 50 \\ 48 \\ \hline 20 \end{array}$$

If we stop after 8, we get 6.8 r.2.
If we continue, we get 3 which keeps repeating.

T: How about N?

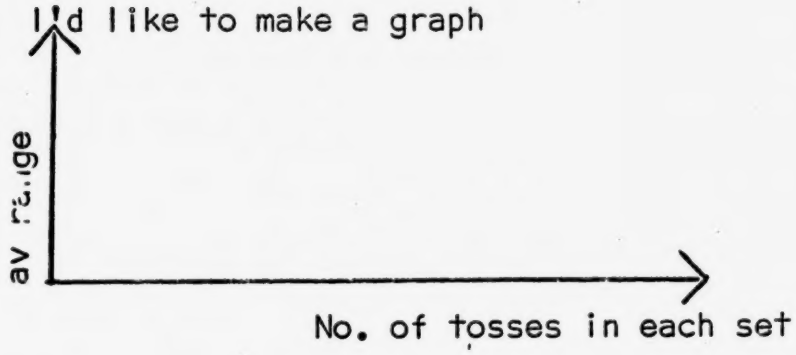
Total N = 76.5

$$\frac{76.5}{6} = 12.75$$

T: I'm going to put these down here (on piece of oak tag.)

	av. r	av. N
T = 5	3	3.3
T = 20	$6.8\overline{3}$	12.75

T: I'd like to make a graph



Mark points on graph.
Explanation of how to mark points.

T = 5, av r = 3

T = 20, av r = 6.833 ...

T: What is happening to our range proportionately in comparison with the number of tosses?
Is it more than 1/2 of the number of tosses?

T: What do you think will happen to the range when we go to T = 45? Is the range increasing at a faster rate or a slower rate than the number of tosses?

Bob: I think it is increasing at a slower rate.

T: What do you think the range might be for T=45?

Bob: I think range will be around 12 or 13.

T: Next time after vacation we will look at T = 45.

Teddy: I think the range will be around 15. So far it's been a little over double.
We multiplied the number of tosses by 4 and the range doubled.

T: I think that is a good theory to go on.
I want to draw a line but let's wait until we have point for T =45.

Teddy: av range will be around 9 for T = 45.
Mult. no. of tosses by 9, mult. av. r. by $\sqrt{9}$

T: Let's look at N, the number which gives us the highest point on the graph.
Graph values for N using zeros

T = 5 av. N = 3.3

T = 20 av. N = 12.75

Does anyone have any ideas as to what r or N will be for T = 45? Are they increasing at the same rate? Teddy?

Teddy: I think it will be 28-1/2.

T: How do you account for that for T = 20 av. N 12.75
Doubling it you get 25.5
You have 5 more and that makes 3 more to get 28.5

Bob: (Questions figures on table)

T: I'll erase the figures for av. r so they won't confuse you.

S: Av N x 2-1/4

T: 3.3 x 2-1/4 = 12.75?

S: No.

S: A little more than 4 times.

Teddy: Double 20 gives you 40. Double 12.75 gives 25.5, 5 more makes 25, 3 more makes 28.5.

T: We will see what happens next time. I was going to do some graphing but let's not, would you like to hear some of the tape back?

S: Yes!

Hancock — 5th Lesson — January 3, 1967

Lee Kubasciewicz

Graphing - as preparation for range and peak graphing kids have discussed graphs of $(2 \times \square) + 3 = \triangle$ and how to change open sentence to make a steeper line.

1. Review graphing truthset of linear equation $(\square \times 4) + {}^{-}3 = \triangle$
   Find open sentences for a steeper line.
   for a less steep line.
   for a horizontal line.
   for a line like \
   Kids came up with questions for both of these.

   Check out coordinates and discuss mult. of neg. nos.

2. Find open sentence for a curve.
   Let them work in pairs.
   If they get a parabola, find ways of turning it upside down, sideways, etc.

   Someone interchanged $\square$ and $\triangle$ in linear equation and made graph.
   Came up with $(\square \times \square) + 1 = \triangle$
   $(\square \times \square) = \triangle$
   $(\square \times \square) - 1 = \triangle$

Asked them to think about how to turn parabola sideways.

Probability - Plans for 6th Lesson on January 12, 1967

1. Pass back results to each team.

2. Record results achieved thus far.

T	av. r.	av. N
5	3	3.3
20	$6.8\overline{33}$	12.75
45		
80		

T = number of tosses in each set.

r = range of numbers of ups.

N = value of the number of ups which had the highest frequency.

3. Organize class to regroup data for T = 45.
Have two teams work together. Elect Chm. (3 teams of 8 students)
Appoint one person on each team to toss thumbtack 50 times.
Appoint another person on each team to record results on sheet for T = 5.

Give each team new sheets for T = 45 and have them find

① find range
② make frequency table
③ make frequency graph
④ find N

4. Get av. r and av. N for T = 45.

5. Make table for T = 80 (use results of 4 teams)

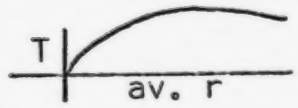
Find r
Make frequency table
Make frequency graph
Find N

6. Record r, N for T = 80 on table

7. Graph results

8. Discuss probability as $\frac{\text{av. N}}{T}$ when T is large

9. Look at relationship between figures in table and slope of graph.

10. Graph 

Fill in table

T	$\frac{av.r}{T}$
5	
20	
45	
80	

11. What would happen if we continued process?

12. Toss two coins - What are expected results - record outcome.

Sixth Session (Lyn McLane January 12, 1967) Hancock 6AP

3 ft. Start

S: Probability mentioned in science - two coins.

T: What are possibilities when you throw two coins?

S: H H
T T
H T

Teddy: More than that - side, side

T: Let's forget about that.

T: If I tossed these two coins 100 times what do you think the outcome would be?

Susan: Chances would be?

Paul: 50-50! The chances are 25 H H, 25 T T and 50 H T.

T: 50-50 for each one Susan?

Paul: 25 H H, . . .

T: Paul I am talking now. If you wish to talk raise your hand.

Susan: 1/3, 1/3, 1/3 I don't know.

David: 33-1/3%

T: If we tossed them many times would we get close to the same amount in each of the three categories?

David: okay

Rob: Chance of 1/3 for each but it could come out in any combination.

Charlie: I disagree with George. Anything could happen.

T: I don't want to get into this right now but maybe when we get through with the thumbtacks. Some of you might wish to toss two coins and record your results.

21 ft. T: Would you arrange your desks as they were last week so you are in teams of 4.
Model for city is on floor - in center of room.
Groups organize on periphery.

T: I've got the graphs from last time which I'll put on the board.

T: Let's keep track of results we have so far

	av r	av N	
T = 5	3	3.3	each person
T = 20	$6.8\overline{33}$	12.75	each team of 4 students

Team with Paul on it had given figures for 3 people because Paul hadn't cooperated.

Let's have Teams I and II Work together
III and IV " "
V and VI " "

T: How many tosses will we get from each pair of teams?

S: 40

T: So in order to get 45 we need to have one person in each group toss thumbtack again. (Appointed tosses)

T. Team I and II Cathy Chairman
III and IV David Chairman
V and VI David R. Chairman

T: The Team chairman are the ones who are responsible for the data of their group.

Handed out sheets for T = 45

T: Would the people who are flipping keep track in the same way you did on the first sheet.

Name _____

Probability

Date January 12, 1967

1. Collect the data from the sheets for T = 20 and from the person in your group is tossing the thumbtack for the second time. Record the data in the table below.

	No. of Ups for Team __	No. of Ups for Team __	No. of Ups for ____ II		Totals
Set 1					
Set 2					
Set 3					
Set 4					
Set 5					
Set 6					
Set 7					
Set 8					
Set 9					
Set 10					
			Total		

No. of Ups □	No. of Sets △
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

2. Find the range. r = _____.

3. Fill in the table to the right to show how many sets had 0 ups, 1 up, 2 up, etc.

   Use graph paper and make a graph of the pairs of numbers in the table.

4. Find N(N is the no. of ups which gives the highest point in the graph). N = _____.

Lee's

January 12, 1967

Cathy	Team 1	Betsy
	Team 2	Cathy
David M.	Team 3	Roger
	Team 4	David M.
David R.	Team 5	David R.
	Team 6	Helena

T=45

		r	N
Cathy	1 & 2	8	30
David M.	3 & 4	11	29.25
David R.	5 & 6	13	33
	Totals	32	92.25

$.\overline{66} \times T = \text{av. } r$ av.r $\frac{32}{3} = 10.666$

$1.5 \sqrt{T} = \text{av. } N$ av. N $= \frac{92.25}{3} = 30.75$

T	Av. r	Av. N
T = 5	3	3.3
T = 20	6.833	12.75
T = 45		

Probability Plans for 7th Lesson January 17, 1967

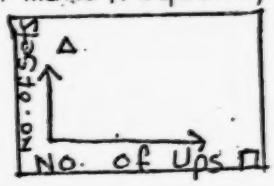
1. Pass back results to Group Leaders

   Cathy Teams 1 and 2

   David M. Teams 3 and 4

   David R. Teams 5 and 6

2. Discuss N and how it is found. Have each team make frequency graph.



3. Make chart on board for T = 45

	r	N
Cathy		
David M.		
David R.		
Totals		

Have Group Leaders fill in results for each team.

4. Find average r and average N.

5. Put table on board and fill in results

	av r	av N
T = 5	3	3.3
T = 20	$6.83\overline{3}$	12.75
T = 45		
T = 80		

av r = $10.6\overline{6}$ av N = 30.75

6. Make table on board for T = 80

   Use results of Team I, II, III, IV

	Team 1	Team 2	Team 3	Team 4		Totals
Set 1						
Set 2						
⋮						
Set 10						

Find r. Make frequency table. Make frequency graph. Find N.

7. Put r, N on table in 5.

8. Graph av N $\frac{\text{---}}{T}$ and find eq. of line.

Seventh Session (Lyn McLane - January 17, 1967) Hancock 6 AP

T: Lot to be done.

First of all. Make frequency graph from information last week.

Second. Find average r and average N and record that on table.

Third. Reorganize data for T = 80.

Explanation of N. as value of number of ups which gives peak on graph. In event that there are several peaks - take average.

Hand back sheets

Students make frequency graphs and find average values for peak and range for T = 45.

Some finished quickly. Miss Kubasiewicz suggested they take out some work for English until rest finished.

Find av. N for T = 45
" av. r for T = 45
} from information recorded on board.

Each team to find total of four numbers for T = 80

Set 1	56	(Teams I, II, III, IV totals for Set 1)
Set 2	53	(" " for Set 2)
Set 3	47	
Set 4	46	

End of tape

Completed table and discussed relationships of numbers.

	Av r.	Av. N
T = 5	3	3.3
T = 20	$6.83\overline{3}$	12.75
T = 45	10-2/3	$30.6\overline{6}$
T = 80	13	51

Had only 1 group of 80.

Ninth Session (Lyn McLane - February 9, 1967) Hancock 6AP

T: What I'd like to do today is to discuss what would happen if we toss a coin. And what would you expect to happen in a coin toss if we tossed it say 10 times? About what ...?

Alec: 5H and 5T

T: If you tossed it 20 times what would you expect?

S: 10H, 10T

T: Suppose we tossed it 30 times

S: I wouldn't expect anything...

T: I'd like you to toss a coin - and after 10 tosses would you write down the ratio of the number of H to the total number of tosses. Suppose you get 6H out of 10 tosses. What is ratio?

S: 12

T: How did you get 12?

S: 6H out 10, 4 tails, 1/2 of 4 is 2 plus the 4 is 6 ...

T: I'm not sure. Supposing I asked you what part of the number of the tosses are heads?

S: 60%

T: or .6, .60, or 6/10. So 6/10 is called the ratio of no. of heads to no. of tosses. Find the ratio of no. of heads out of 20 tosses, 30 tosses, ... 100 tosses.

T: Does anyone have any idea what might happen to these ratios?

T: Maybe best way is to toss coin up, catch it in hand and flip it over.

Class starts tossing. Some figured out ratios for each set of 10 instead of finding ratios for no. of heads out of 20 tosses, 30 tosses, etc.
They found decimal names for ratios.

T: Could sheets be collected tomorrow? (to Miss Kubasiewicz)

T: How many are almost finished?
(Some are still tossing.)

T: Would you please finish them. I would like to look at the decimal names - Do they get closer to .5 or do they get further away from .5?

	Av. R	Av. N
T = 5	3	3.3
T = 20	6.833	12.75
T = 45	10.666	30.666
T = 80	13	51

	R	N
Cathy	8	30
David M.	11	29
David R.	13	33
Totals	32	92

Av. r = 13
Av. N = 30.666

Graph (Freq. for T = 80) "Looks like a church"

Consensus of opinion = 50% chance of ups and downs.

Discussion - What happens to the range when T increases?

Bob - R increases by 3.

What happens to the range as compared to range of tosses?

David R. Go down.

* Teddy 50% - 50% chance but according to graph ups will increase as number of tosses increase.

Teddy was predicting value of N for T = 1000.
He also suggested predicting for T = and T = after pattern in table.

Teddy thought graph for N ——————

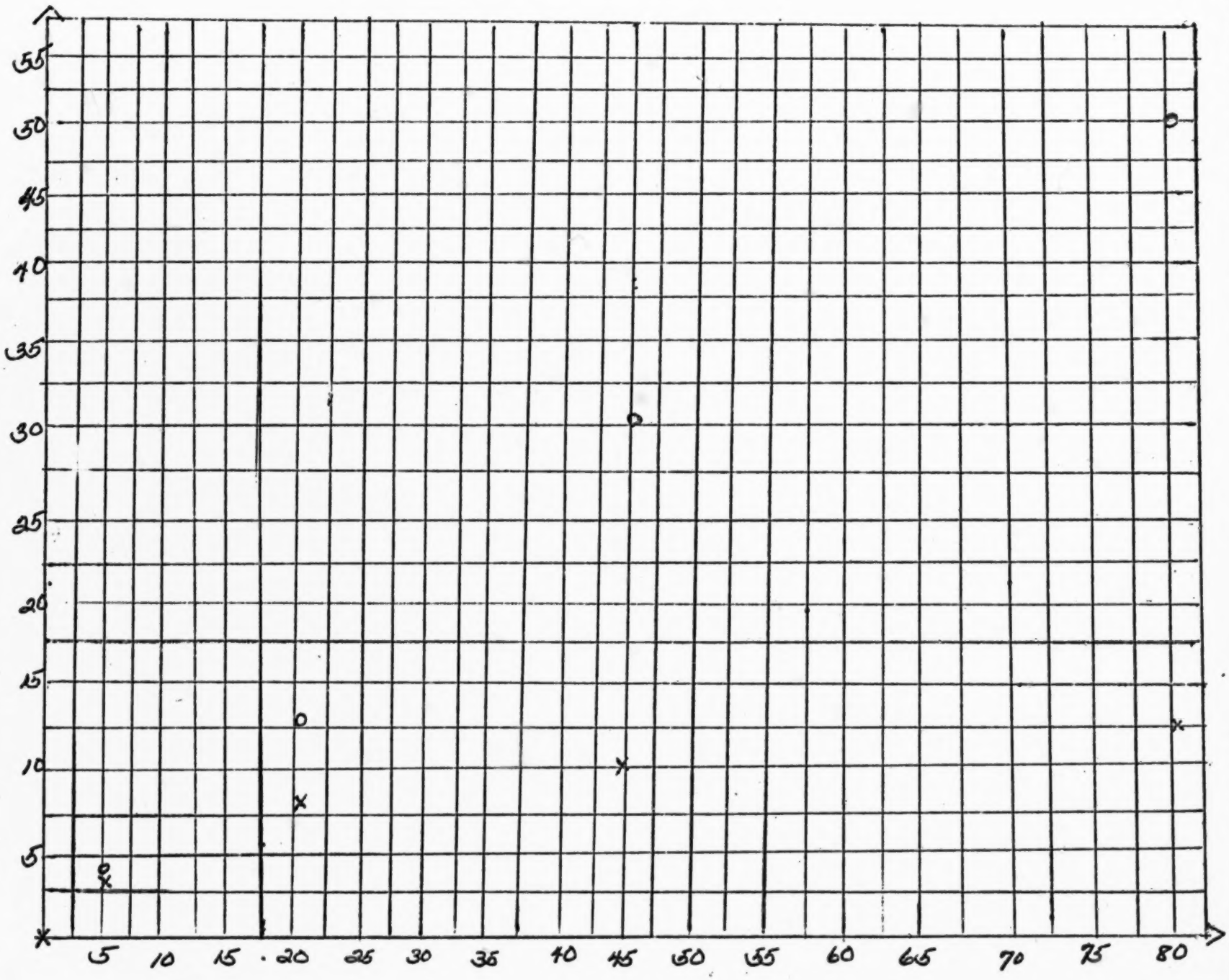
graph for r ———————

Said we would graph next time

David M. offered to throw tack 1,000

Look at die next time.

0 for av N
x for av r



Plans for Probability Lesson on February 13, 1967, Tenth Lesson

(Look at sheets of last Thursday and maybe organize results for a class discussion on the change in the ratio of the number of heads to the number of tosses as the number of tosses increases.)

Pose question - what is the probability of getting two heads when tossing two coins?

What is probability of getting at least one head?
Set of possible outcomes by Cartesian product.

Discuss probabilities with three coins -

Dice :

Cards

Blaise Pascal 17th Century mathematician developed probability for use in gambling.

Graphing - st. lines - parabolas

References on Probability

Kline, Morris, Mathematics in Western Culture New York: Oxford University Press (1953).

Adler, Irving Mathematics: The Story of Numbers, Symbols and Space New York: Golden Press (1961).

Newman, James R. The World of Mathematics New York: Simon and Schuster (1956).

187 ft.

Dr. Lomon: Okay I'm going to watch how they spin in case there is some difference. You both bet 5 so you got zero and they get 4.
All right, have you decided on your bets?
Window team bets > 7 T
Door team bets > 5 T

Okay, spin.

Dr. Lomon: What is it?

5 T
7 T

David: Neither of us won.

Dr. Lomon: Neither of you won so both teams get zero. You will continue this with Miss McLane but why don't we use the few minutes which are left for you to ask me some questions. Would everyone please move back to there.

Dr. Lomon: I have just added up. Would someone please check for me. We have here 12 trials - thats 120 tosses.
66 T, 44 heads.

S: The way you added up the heads and tails it's 110?

Dr. Lomon: So it's worse than that, its 76 T. I didn't add up the tails? Why didn't I add up tails?

S: You added up a heads and you knew how many all together.

Dr. Lomon: Does this final answer surprise you in any way? It's not the final answer because you will go on tossing. But does this answer surprise you? No? Yes?

S: No because you don't know what you will get in the end.

Dr. Lomon: Does one ever learn anything about what the averages are, because if you have to wait until you do an infinite number of problems, you never finish. But does one ever learn anything about what the averages are or does one never know? David?

David: You learn a little bit but not all .

Dr. Lomon: Bob?

Bob: I don't think so because there hasn't been equal,you know, one more than the other ...head, I've never spinned it like this before. More tails.

Dr. Lomon: But we are doing something d'fferent. Well look you say that you don't learn anything, yet your betting has shown that you did learn something - you are doing something in your betting which indicates that you believe something is happening. After all if you thought it was really 50-50 and I think it's a debate, then why did you bet $>$ 6, $>$ 7 and always $>$ 5? George?

George: The Door Team, it's always an average of 5 - the betting each time.

Dr. Lomon: The betting yes - more than 5 not 5 you see after all, half of them would be 5 and they are always betting on more than half. Jef?

Jef: Well, we both started out betting about 50-50, then when we got more tails than heads all the time, people started betting more tails.

Dr. Lomon: Yeh, that's what I would say would happen and I would say that makes some sense. But that says uou are finding out something. That even though we haven't done an infinite number of tosses, we are beginning to get some idea of what the average is going to be. Now actually with the thumbtack experiment you learned some things the same. The real question is, I think it was David brought it up, the real question is we have learned something, but how much have we learned? How sure are we going to be. For instance, do we think it is going to be mostly between 5 and 8 or mostly between 6 and 7 or do we think mostly it's going to fall half the time between 4 and 9 you see. That kind of question has to come up. What is the range if you remember that word from the thumbtack. Well, we will look into that later but we'll probably go on betting a bit more before we go into that. Well it's time for your milk now, so unless you have any questions to ask I think it is time to end our talk. Any questions?

S: What about our lists?

Dr. Lomon: Maybe you better give them to your teacher to keep for next time.

Oh. Do you think these coins are in any way loaded? If things aren't really 50-50, why? What is it about the coin which makes a difference when you spin it and doesn't make a difference when you toss it? Yes?

S: Well,

S: There might be more ...?

David - (Something about way of spinning)

Dr. Lomon: But if we always spin coin the same way what is there about the coin which makes it come up more tails?

David: (Something about head side being heavier)

Dr. Lomon: Think about this. The design: on the tail side and head side is different.

Name _____

Date _____

Resuits of Spinning Nickels

March 2nd

Bets		Number of Tails in Ten Spins		Points	
Window	Door	Window	Door	Window	Door
> 4T	>4T	6	6	2	2
> 5T	>5T	7	6	4	4
> 5T	>5T	9	7	4	4
> 6T	>6T	8	3	0	6
>5T	>5T	8	4	0	4
>7T	>5T	5	7	0	0

March 6th

>4T	>5T	5	8	9	7	5	8	2	4	8	6	6	5	8	16
>6T	>5T	3	6	7	8	6	6	6	8	9	8	7	6	24	20
>6T	>5T	5	7	7	4	3	7	4	6	6	4	4	5	0	12
>5T	>5T	1	5	3	9	4	5	5	9	7	8	8	4	16	4

Above are the results obtained thus far in the game of spinning nickels. The scoring system is given below. Use the above data and the scoring system to figure out the most advantageous bet your team should make next time.

> 0T-1/10 point, > 1T-1/4 point, > 2T-1/2 point, > 3T-1-1/2 points, > 4T-2 points,

> 5T-4 points, > 6T-6 points, > 7T-8 points, > 8T-11 points, > 9T-14 points.

Twelfth Session (Lyn McLane - March 6, 1967) Hancock 6AP

T: What we need to do this morning is to continue the game we were doing last time and I think last Thursday with Dr. Lomon the Window Team had 10 points and the Door Team had 20 points. Here is the same chart we had up.
I'd like to have you work in pairs - one person will be the recorder. The Window Team will bet for all six. Some you will win on and some you won't.

(Made chart on board)

S: Not enough people to record.

T: Then one person will have to record for two people.

T: You will have to spin on your desk top. What happens if the nickel hits the pencil strip?

S: Doesn't count.

T: You will have to get good at spinning. Let me know when you are ready to place your bets.

T: What is your bet Betsy?

Betsy: 4T

T: Window bets more than 4T, and the Door bets more

S: than 5T.

T: 8 points for Window.

T: Are you done yet? Betsy why don't you just read them.

Betsy: 5T, 8T, 9T, 7T, 5T, 8T

T: You won 4 of them that's 16 points for the Door Team. Are you ready to bet again?

Betsy: > 5T

(They reconsider since other team wasn't ready.)

T: Window bets > 6T, Door bets > 5T.

Bob: 6 8 9 8 7 6

T: How many points?

S: 24

T: That's 6 x 4, 24 for the Window Team.

Betsy: 3, 6, 7, 8, 6, 6

T: Paul would you keep going? How many did you win on?

S: 5.

T: That's 4 points for each one - 20

T: Betsy?

Betsy: > 6T

T: Bob?

Bob: > 5T

T: Bob what do you have?

Bob: 4, 6, 4, 4, 4, 5

T: Let's wait until Paul finishes.

T: How many points do you have?

S: None

T: None for the window

T: I don't think we have time for another.

Discussion

T: All right. Let's do another real quick
> 5T, > 5T (bets)

Bob: 5, 9, 7, 8, 8, 4

Betsy: 1, 5, 3, 9, 4, 5

T: Would you all sit down please.

T: What I'll do is to put information on ditto sheet.
Before I come back I'd like some of you to make frequency table and then we'll make graph like we did for the thumbtacks.

(Figured out total score - some disagreement)

T: Get back in your own seats and get ready for next class.

Sixteenth Session (Lyn McLane - April 13, 1967) Hancock 6AP

Discussion of tape recorder and listening to tapes.

Gave out graph paper and ditto sheet on graphing, straight lines and parabolas.

T: Let's stop at 20 after to discuss our results.

Some students had trouble getting started on finding first pair of numbers which would go in open sentence. The first problem was to graph $\square = \triangle$. I think many of them thought that it was not allowable to put the same number in $\square$ and $\triangle$.

Probability - Plans for Seventeenth Session

Lyn McLane - April 24th, 1967

Find out who did more spinning.

Read problem out of Gardner's book.

Joe says: I will give you a dime if 3 coins land all heads or all tails. You give me a nickel if they land otherwise. Do you accept the bet Jim? Should Jim accept?

If there is new nickel data - start compiling for S = 90 and S = 160 (Use old data too) (Graphing or something for rest of class.)

If there is not enough new nickel data - have class break into two teams again with each having 6 spinners.

The class has to watch TV Show at 9:30 - So just work on three Penny Problem out of Gardner's book.

H H H	T H H	$\frac{1}{4}$	$\frac{3}{4}$
H H T	T H T		
H T H	T T H		
H T T	T T T		

Results:

April 24, 1967

Charlie had feeling that 3H or 3T would not come up very often.
I had to pull to get listing of 8 possibilities.
Some objected to ordering - they thought there were only 4 possibilities.

2H	1T
2T	1H
3T	
3H	

Class was slightly more in favor of 8.
Others argued its 50-50 whether or not you get 2T or 2H on 1st 2 coins and 3rd is 50-50.

Did quick tossing

				Totals
2H	1T	8	3	11
2T	1H	6	3	9
3H		3	2	5
3T		4	0	4
				29

Probability Plans for 18th Session - May 4th 1967

Hancock 6AP

L. McLane

If Teddy has his data for 1000 nickel spins in - discuss estimates for range and peaks for S = 160, as compared with values obtained from data. Were our estimates close? Mark values on graph.

Bring up discussion of last time. Results of quick tosses last time

2H	1T	1H 2T	3H	3T
11		9	5	4

What were two sides of the argument?
How to find resolution: :

With one coin. With two coins. With three coins.

Teddy and Kim tossed coins and ended up with

27 TT 27 HH HT 23 TH 23

Maybe make tree.

Have everyone toss 3 coins or 1 coin 3 times.

If time, put graphs on board and have class try to guess open sentence.

Results: Dr. Lomon came to observe.

Used data on Teddy's sheet - not much and plotted range and peak.

Susan suggested that maybe we ought to try a different line (peak) than the one we used last time.

With more data, Bob suggested that range might increase - but not much.

Get Ten sets of 10, 40, 90, 160
See how range and peak look.

Continuation of argument of last time. Looked at results from last time and found that they didn't fit 4 possibility theory.

Look at 3 to 1 bet.

3 made up of 1 + 1 + 1

If eight different cases

What is set of all possible things which can occur? HTH, etc.

What is subset of things all of a kind? (HHH, TTT) Chance?

What is subset of things not all of a kind? Chance?

Would anyone care to make up another good bet involving another subset? 2H (50-50 bet)
Anything with heads (7 to 1 bet)

Probability Plans for 19th Session - May 8th 1967

Hancock 6AP L. McLane

Look at 7 groups each for S = 10, 40,90, 160 and plot range and peak.

Discuss three coins of last time.

Results:

Charlie had more data - which gave enough for 8 groups of 160. So we looked at 8 groups in each Sample Size.

S = 10 6, 6, 7, 6, 9, 7, 8, 3

S = 40 25, 27, 24, 29, 19, 25, 24,26

S = 90 60, 58, 47, 64, 49, 40, 54, (3+6+6+5+6+5+4+6+7)

S = 160 86, 78, 82, 77, 105, 94, 92; (8+7+2+6+6+5+6+5+6+8+1+5+3+9+4+5)

Found range and peak

	Range	Peak
S=10	6	6
S=40	10	24.5
S=90	24	52.5
S=160	28	86

Plotted points on overhead - some were different than what we had before.

Susan fit line for peak points.

Betsy drew curve for range points - sort of like parabola.

Suggested that for next time they think about making fair bets --

David's definition of a fair bet -- when each person has a 50-50 chance of winning.

There was disagreement as to the fairness of bet discussed last time.

For Teddy's Data

May 8, 1967

Number of Tails in 10 spins of nickel (Sets of 160)

From ditto sheet

25+27+24+29 = 105

19+25+24+26 = 94

30+23+20+19 = 92

18+23+27

5	2	6/54	3	6	6	5	6	
						Rogers		
5	4	6	7	7	6	6	6	86
6	4	7	3	3	4	7	6	
4	5	5	7	3	6	4	4	78
4	6	6	7	4	5	6	7	
3	4	6	5	4	3	6	4	82
4	7	6	3	5	4	7	4	
5	6	7	3	3	4	3	6	77
6	6	7	6	9	7	8	3	
8	4	5	7	5	8	9	7	105
5	8	2	4	8	6	6	5	
3	6	7	8	6	6	6	8	94
9	8	7	6	5	7	7	4	
3	7	4	6	6	4	4	5	92

No of Tails in

Sets of 90 — 60, 58, 47, 64, 49, 40, 54

Sets of 40 — 25, 27, 24, 29, 19, 25, 24

Sets of 10 — 6, 6, 7, 6, 9, 7, 8

Probability - Plans for 20th Session — May 18, 1967

Hancock - 6AP — L. McLane

1. Before we can work on making fair bets, we need to come to an agreement on fairness of bet with three coins.

2. How could we reach an agreement?

   a. So far verbal arguments do not seem to do the trick.

   b. Experimentally?

      How to conduct experiment?

      Toss one coin 3 times?
      Toss 3 coins 1 time?
      } Repeat how many times

      Record results as H T H with order assigned?
      or record results as 2H 1T with no order?

d'Alembert 18th Century French mathematician

Two coins 3 outcomes

Laplace - Two coins - 4 outcomes

3. Conduct experiment

4. Discuss results

Results: See accompanying sheet. Not many changed their minds even though data was in strong support of Laplace.

Hancock 6AP May 18, 1967

Probability

Votes
- 14 d'Alembert $\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$
- 4 Laplace $\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}$

Everyone in class tossed two coins and recorded data whichever way they wanted.

Method A

		Totals	
H H	3, 8, 7, 6, 7, 6, 6, 7, 3, 5	58	111
T T	1, 8, 12, 9, 3, 4, 5, 7, 4, 6	59	104
H.T T.H	1, 8, 20, 12, 17, 13, 8, 7, 8, 10	104	208
		221	

Method B

H H	10, 11, 7, 10, 7, 5, 3	53
T T	3, 10, 12, 8, 2, 5, 5	45
H T	5, 5, 12, 15, 7, 5, 1	50
T H	7, 11, 10, 11, 7, 5, 3	54
		202

Votes
- 13 d'Alembert
- 6 Laplace

David R. $\frac{1}{4},\ \frac{1}{4},\ \frac{1}{2}$

T: I agree with you - really Laplace.

Probability - Plans for 21st Session — May 22, 1967

Hancock 6AP — L. McLane

Discuss 3 coin problem

What is likely to happen when we toss 3 coins many times? What is probability of getting 3T? 2H 1T?

If answers are correct then "what do you think should happen?"

Pass out sheet listing choices for project.

Discuss each one briefly except 6 which should be discussed in depth. (A model of how to do experiment) - Maybe add to list.

Choose one of the following. Instructions are on the next page.

1. 2 dice - 400 tosses
2. 1 die, 1 coin - 150 tosses
3. 2 thumbtacks - 100 tosses
4. 3 coins - 100 tosses
5. 3 thumbtacks - 100 tosses
6. 2 dice, 1 coin - 750 tosses

	Peak Av. N	Av. N / T
T=5	3.3	.66
T=20	12.75	.6375
T=45	35.75	.7944

$3\overline{)2.0919}$

.6973

P(1H) P(1T) P(6) P(2,2) P(2,2,H) P(6,2,T) P(66) P(2,6,T)

individual probabilities

Use .7 as P(U)

.3 as P(D)

7. 1 coin, 1 thumbtack - 100 tosses
8. 1 die, 2 coins - 150 tosses
9.
10.

Plans (cont.) May 22, 1967

Instructions:

1. Which item did you choose? _____
2. What are the individual probabilities? _____
3. What do you think are the possible outcomes? _____

   _____

4. What do you think is the probability for each outcome? _____

   _____

   _____

5. Conduct an experiment and list the results of the indicated number of tosses. (You may do more tosses if you so desire.)
6. Be preposed on Thursday, June 1st to discuss* your experiment with the whole class and to hand in a copy of your results.

* If you would like to have an overhead transparency of your results to use in the class discussion, put a copy (typed or in pencil) of your results on my desk on the 3rd floor by Wednesday, May 31st and I will have a transparency ready for use in class on Thursday.

Results: May 22, 1967

Alison brought in clipping from yesterday's N.Y. Times about Penny Flipping - out of 300,000 or more 50.2% heads. Several people in class felt that .2% was not large enough to draw conclusion that heads are favored.

She also had Vol. II of the The World of Mathematics which has section on probability. Teddy asked if they had results or was it just talk.

Three coins —

discussed individual prob. $P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$

Possible outcomes	guessed Probabilities listed by students			
3H	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{6}$
3T	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{6}$
2H, 1T	$\frac{4}{10}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{2}{6}$
2T, 1H	$\frac{4}{10}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{2}{6}$

Discussed $P(5) = \frac{1}{6}$, $P(5,5) = \frac{1}{12}$, $\frac{1}{36}$, $\frac{1}{18}$

David R. Tommy:

Passed out sheets. Tommy wanted to do 9. Two dice, 1 Tommy's die (4 u's, 2 p's) - 1000 tosses.
Charlie asked about how to record data.
Discussed 6) one coin, 2 dice.

Individ. Prob.		Possible Outcomes	
$P(1) = \frac{1}{6}$	$P(H) = \frac{1}{2}$	1,1,H	
	$P(T) = \frac{1}{2}$	2,1,H	1,2,T
		⋮	⋮
$P(6) = \frac{1}{6}$		3,1,H	⋮

Plans for 22nd Probability Lesson — June 1, 1967

Listen to reports.

Results:

The following had reports to give:

Cathy	Susan
Teddy	David R.
Karen, Alison	Roger
Kim	Helena

We heard the first four - the last four and Tommy Spencer (from Mrs. Delsey's room) will talk on Monday.

They were all confused about the individual probabilities but otherwise did a good job on their reports.

Cathy did 2. One die, one coin

Teddy did 1. Two die

Karen, Alsion did 1. Two die

Kim did 3. Two thumbtacks

On all of them we found the individual probabilities and the number of possible outcomes. We also found the expected frequency of each outcome and compared it with the actual frequency.

We discussed Teddy's at some length -

What is P(a,a)? $a \in \{1, 2, 3, 4, 5, 6\}$

What is P(a,b)? $a < 3$ and $b < 3$

What is P(a,b)? $a + b = 7$

The 3rd partner with Karen and Alison did not do her duty so they ended up with fewer than 400 tosses. They said theirs was the same as Teddy's but with smaller numbers.

Kim was reluctant to do his because he did not understand the individual probability question. We discussed this with the whole class using $P(u) = \frac{7}{10}$ and $P(s) = \frac{3}{10}$. S=Side. Possible outcomes were U,U U,S S,S S,U which is more likely to happen U,U or S,S? Most agreed on U,U. Kim felt that S,S would occur more often than (S,U) - it had in his experiment. But others disagreed - they said $P(U) > P(S)$ $\therefore$ $P(S,U) > P(S,S)$.

Different people assigned different probabilities to possible outcomes.

P(U,U)	$\frac{7}{10}$	$\frac{6}{10}$	$\frac{1}{4}$	$\frac{13}{20}$	$\frac{6}{10}$
P(U,S)	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{1}{4}$	$\frac{3}{20}$	$\frac{1}{10}$
P(S,S)	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{1}{4}$	$\frac{1}{20}$	$\frac{2}{10}$
P(S,U)	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{3}{20}$	$\frac{1}{10}$

Looked back at individual prob. from first three examples and tried to find out how to derive the probability for each possible outcome. Multiplication of individual probabilities worked so we tried it for the thumbtack and got $P(U,U) = \frac{49}{100}$

$$P(U,S) = \frac{21}{100}$$

$$P(S,S) = \frac{9}{100}$$

$$P(S,U) = \frac{21}{100}$$

which did not agree with Kim's data very well.

Next time-Reports from Susan, David R., Roger, Helena, and Tommy Spencer.

Cathy's data for 150 tosses of one die and 1 coin $P(1) = \frac{1}{6}$, $P(2) = \frac{1}{6}$, . . ., $P(6) = \frac{1}{6}$ $P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$.

Probability (cont.)

12 possible outcomes - with probability of $\frac{1}{12}$ for each.

	1	2	3	4	5	6	
T	13	14	12	15	10	12	80
H	12	12	10	7	11	13	70
	25	26	22	22	21	25	

$\frac{1}{12} \times 150 = 12\frac{1}{2}$

Teddy's data for 400 tosses of two dice.

$P(1) = \frac{1}{6}$, . . ., $P(6) = \frac{1}{6}$

36 possible outcomes - with probability of $\frac{1}{36}$ for each

1st Die \ 2nd Die	1	2	3	4	5	6
1	9	5	8	12	9	6
2	9	11	15	13	13	11
3	9	9	3	11	13	12
4	7	11	11	11	11	10
5	8	9	10	21	19	6
6	19	10	18	16	19	13

$\frac{1}{36} \times 400 = 11.\overline{1}$

Probability (cont.)

Karen - Alison's data for 262 tosses of two dice.

$P(1) = \frac{1}{6}$, . . . , $P(6) = \frac{1}{6}$

36 possible outcomes - with probability of $\frac{1}{36}$ for each.

2nd Die

1st Die	1	2	3	4	5	6
1	13	5	6	5	6	6
2	6	5	9	11	9	6
3	6	5	9	8	6	5
4	8	12	9	8	2	4
5	4	1	11	12	11	6
6	5	6	4	7	6	10

(They did not get all of them but did not have time to check)

$\frac{1}{36} \times 262 = 7\frac{5}{18}$

Kim's data for 100 tosses of two thumbtacks.

$P(U) = .7$ $P(S) = .3$

U,U	33
S,S	24
S,U	20
U S	23

$P(U,U) = .49$

$P(S,S) = .09$

$P(S,U) = P(U,S) = .21$

[Dr. Lomon suggested that maybe S,S occurred so many times because the thumbtacks were thrown together.]

Plans for 23rd Probabiiity Lesson

June 5, 1967

Hancock 6AP

Miss McLane

1. Ask Kim if he threw thumbtacks together.

2. Reports from

   Sue
   David R.
   Roger
   Helena
   Tommy

Results:

Kim did throw thumbtacks together but the tacks did not touch each other. I suggested that maybe that would explain why S,S came up so often. (Didn't mention pinball machine with twin boards but single pull because I wanted to get reports done before whole school went off to Assembly.)

David R's results of tossing 3 coins 100 times.

$P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$, $P(H,H,H) = \frac{1}{6}$, $P(T,T,T) = \frac{1}{6}$, $P(2H, 1T) = \frac{2}{6}$

$P(1H, 2T) = \frac{2}{6}$

H,H,H	12
T,T,T	12
2T, 1H	33
2H, 1T	43

$\frac{1}{6} \times 100 = 16\frac{2}{3}$

We discussed method of finding expected probabilities by multiplying which we used last time. Applying that method to David's problem we came up with $P(H, H, H) = \frac{1}{8}$

$P(T,T,T) = \frac{1}{8}$

(continue on next page)

Probability (cont.) June 5, 1967

(I think David figured these out by taking $\frac{1}{2} \times \frac{6}{8}$.)

$P(2T, 1H) = \frac{3}{8}$

$P(1T, 2H) = \frac{3}{8}$

$\frac{1}{8} \times 100 = 12\frac{1}{2}$, $\frac{3}{8} \times 100 = 37\frac{1}{2}$

David felt that his data was closer to the latter calculations. He thought that if he counted the number of times HTH occurred it would be around 12.

Roger's results of 400 tosses of two die.

$P(1) = \frac{1}{6}$, ..., $P(6) = \frac{1}{6}$.

$P(1,1) = \frac{1}{36}$, $P(1,2) = \frac{2}{36}$, $P(2,1) = \frac{2}{36}$, $P(1,3) = \frac{3}{36}$, $P(3,1) = \frac{3}{36}$.

[Roger added the numbers on the dice and assigned to that pair the probability of getting that sum - i.e. $P(1,3) = \frac{3}{36}$ because 1 + 3 = 4 and P (sum of 4) = $\frac{3}{36}$.]

Susan objected to assigned probabilities because their total was more than 1. David R. said that $P(1\ 1) = \frac{1}{6}$ $P(1,2) = \frac{1}{6}$ etc.

1st Die

	1	2	3	4	5	6
1	10	16	9	12	11	12
2	8	6	7	14	7	8
3	12	8	11	5	8	6
4	13	10	13	16	10	7
5	10	10	11	7	17	14
6	13	10	12	16	13	13

Probability (cont.) June 5, 1967

Results of Sue's tossing 1 die and 1 coin 150 times $P(1) = \frac{1}{6}$, . . ., $P(6) = \frac{1}{6}$ $P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$ $P(1,H) = \frac{1}{12}$ because

there are 12 possible outcomes.

	1	2	3	4	5	6
H	10	14	11	20	11	16
T	12	4	12	12	15	12

$\frac{1}{12} \times 150 = 12\frac{1}{2}$

The highest number is 20 20 - 12 = 8

The lowest number is 4 12 - 4 = 8

Cathy: That's a coincidence.

Tommy Spencer was unable to come to class because of a rehearsal, but we discussed the problem he was going to do. Toss 2 regular dice and 1 "Tommy's die" which had 4 u's and 2 p's.

Cathy and Stephen said $P(U) = \frac{4}{6}$, $P(p) = \frac{2}{6}$

T: How many possible outcomes?

S: a, 48, b, 44, c, 72, (Students started listing possibilities)

T: Who can make a fast speech to convince us?

Sheila: 6 X 2 X 6 = 72

for regular die for Tommy's die for regular die

Last class on June 15th.

Tommy and Helena.

Name _____

Date _____

Choose one of the following, then turn to the next page for instructions.

1. Two dice - 400 tosses
2. One die, one coin - 150 tosses
3. Two thumbtacks - 100 tosses
4. Three coins - 100 tosses
5. Three thumbtacks - 100 tosses
6. Two dice, one coin - 750 tosses
7. One coin, one thumbtack - 100 tosses
8. One die, two coins - 150 tosses
9.
10.
11.
12.

Name _____

Instructions

1. Which item did you choose? _____

2. What are the individual probabilities? _____

_____

3. What are the possible outcomes? _____

_____

_____

4. What do you think is the probability for each outcome? _____

_____

_____

5. Conduct an experiment and list the results of the indicated number of tosses. (You may do more tosses if you so desire.)

6. Be prepared on Thursday, June 1st to:

   a) discuss your experiment with the whole class

   b) hand in a copy of your results.

L. McL.
Hancock

Name _____

Date _____

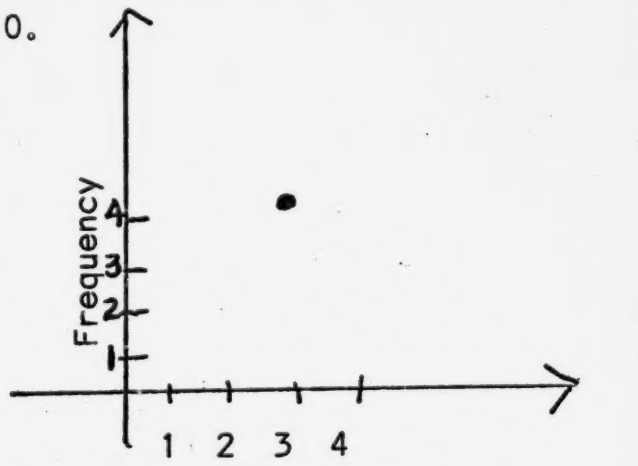
Select one of the following methods to make an analysis of the results obtained in the Nickel Spinning Game. When you are finished, look at the graphs you have made and the point system and figure out which bet is likely to be the most advantageous for your team.

Method I.

This method is similar to the analysis make for the thumbtack experiment. Let S = number of spins.

A. Make a frequency table and graph for S = 10.

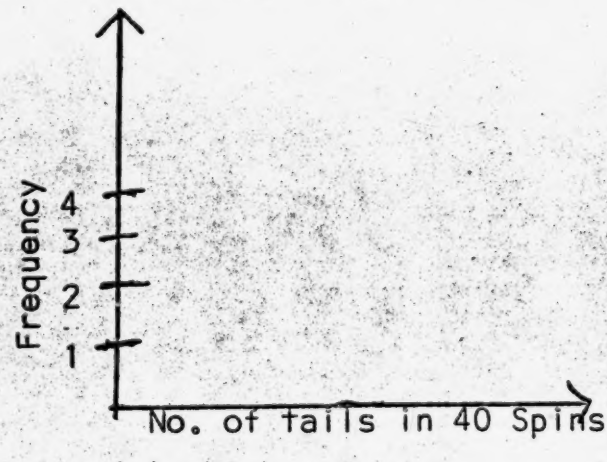
No. of tails in 10 spins	No. of occurrences (frequency)
0	
1	
2	
3	4
4	



Find the range (r) and the no. of tails (n) which occurs most often.

P. Make a frequency table and graph for S = 40. Obtain the number of tails for each group of 40 by adding up the number of tails for the first 4 sets of 10 spins (6+6+7+6), for the second 4 sets of 10 spins (9+7+8+3), etc. Read horizontally rather than vertically.

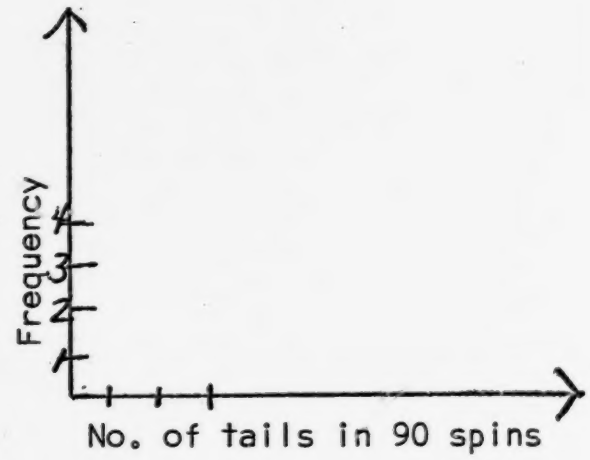
No. of tails in 40 spins	Frequency
17	
18	
19	2
20	
.	
.	



Find the range (r) and the number of tails (n) which occurs most often. (If the frequency is the same for several numbers, let r be the average of those numbers.)

C. Make a frequency table and graph for S = 90. Obtain the number of tails for each group of 90 by adding up the number of tails for the first 9 sets of 10 spins (6+6+7+6+9+7+8+3+8), for the second 9 sets of 10 spins (4+5+7+5+8+9+7), etc.

No. of tails in 90 spins	Frequency



Find the range (r) and the number of tails (n) which occurs most often.

D. Fill in the table below:

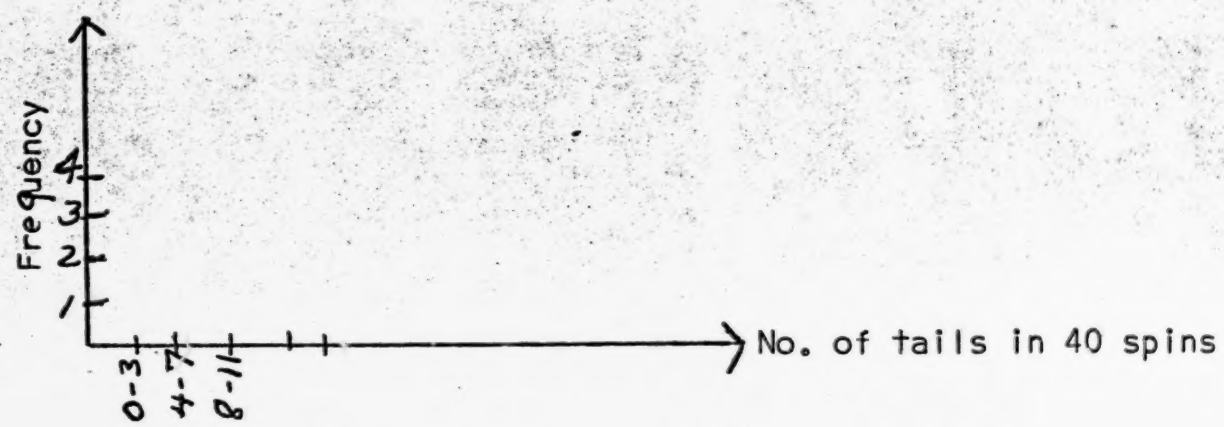
	r	n
S=10		
S=40		
S=90		

Method II

Follow the instructions in Method I for making the frequency table and finding r and n. Two of the frequency graphs will be made differently.

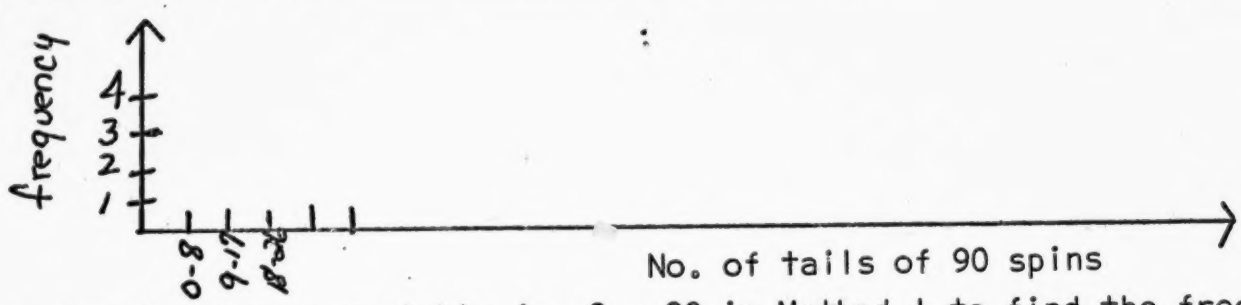
A. For S = 10 make the graph as in Method I

B. For S = 40 Label the horizontal axis as in the figure below:



To mark points on the graph, look at the frequency table for S = 40. Observe that the frequency for 12 heads, 13 heads, 14 heads, 15 heads is zero. On the graph above 12-15 mark a point to indicate zero frequency. The frequency for 16 heads is 0, for 17 heads is 0, for 18 heads is 1, for 19 heads is 2. Therefore, the frequency for values from 16 to 19 is 3. Mark a point on the graph above 16-19 to indicate a frequency of 3. Continue in a similar fashion.

C. For S = 90, Label the frequency graph as follows:



Use the frequency table for S = 90 in Method I to find the frequencies for values from 9 to 17, 18 to 26, etc Mark the corresponding points on the graph. How do these graphs differ from those made in Method I?

Method III

Use this method only if you have an ordered record of the outcome of each spin. For example, your record might look like this:

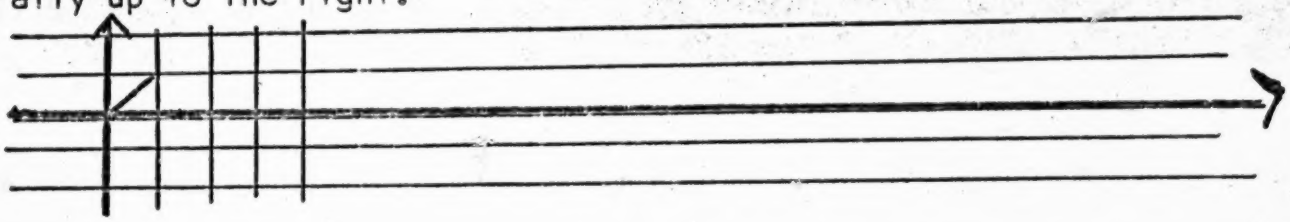
H T T T H T H T T T.

T T H H H T T H T T

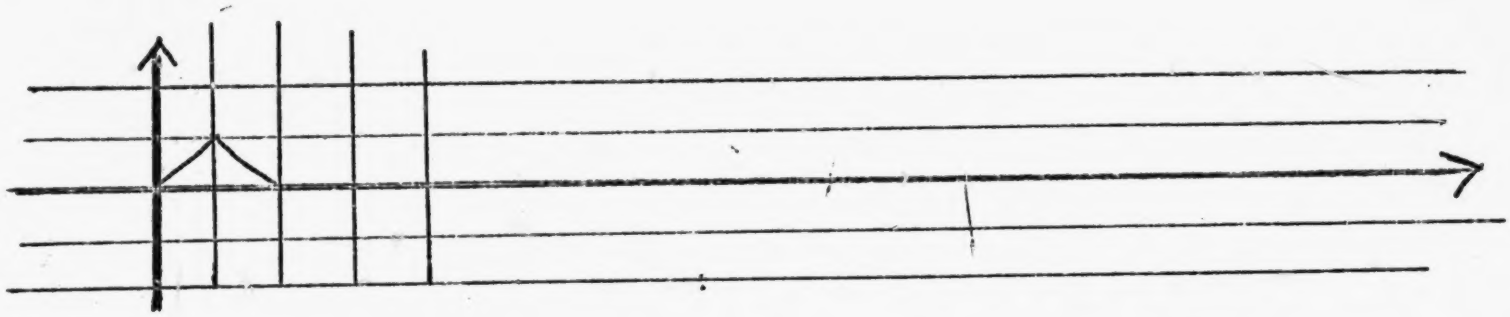
ETC.

A. On a piece of graph paper make the axes as illustrated:

B. Start at the origin (the intersection of the axes). Look at the first outcome H (in the sample record above). Move diagonally up to the right.



Look at the next outcome T. Move diagonally down to the right.



Continue making this "random walk", moving diagonally up to the right if the outcome is H and diagonally down to the right if the outcome is T. You will probably need to continue your "random walk" on another piece of graph paper.

C. When you have finished, look at the graph paper and answer the following questions:

1. How many times does the path cross the horizontal axis?
2. Is the path more above the horizontal axis than below?
3. How many times did 4 heads occur in succession?
4. Did the path come out as you expected it would?

Method IV

This method is similar to Method I but only a portion of the results is used for S = 10. What can be learned from the data if only 15 groups of 10 spins each, 15 groups of 40 spins each and 6 groups of 90 spins each are considered.

A. Look at the results of the first 15 sets of 10 tosses. Make a frequency table and a frequency graph for S = 10 using only those 15 figures. Find the range (r) and the number of tails (n) which occurs most often.

B. Use the procedure described in Method I to obtain 15 sets of 40 tosses. Make a frequency table and frequency graph. Find r and n.

C. Follow the instructions for part C in Method I.

D. Fill in the following table:

	r	n
S=10		
S=40		
S=90		

How do you think this table will differ from the table obtained in Method I?:

Data Collected during Probability Unit

by

Members of Sixth Grade AP

Hancock School

Lexington, Massachusetts

Regular Teacher: Miss Kubasciewicz

Math Specialist: Miss McLane

June 22, 1967

Probability Unit
Sixth Grade AP
Classroom Teacher:
Miss Kubasciewicz
Special Teacher:
Miss McLane

Hancock School
Lexington, Mass.
Dec. 8, 1966

THUMBTACK EXPERIMENT

Frequency Table for the Number of Ups for Each Pupil*

		Number of Ups					
		0	1	2	3	4	5
Team 1	Betsy	0	1	1	4	3	1
	Nancy	0	0	1	7	2	0
	Susan	0	1	1	4	3	1
Team 2	Bob	0	0	1	5	1	3
	Cathy	0	0	1	2	5	2
	Sheila	0	0	4	3	1	2
	Jody	0	0	2	2	4	2
Team 3	Roger	0	3	0	4	3	0
	Brian	0	0	2	3	3	2
	Stephen	0	0	2	1	7	0
	Jef	0	1	1	3	5	0
Team 4	David M.	0	0	4	1	5	0
	Charlie	0	1	2	2	2	3
	Kim	0	4	1	2	2	1
Team 5	David R.	0	1	5	3	0	1
	George	0	1	3	2	2	2
	Susan B.	0	0	1	2	2	5
	Karen	0	0	5	2	3	0
Team 6	Alison	0	1	1	3	3	2
	Alec	0	0	2	8	0	0
	Helena	0	0	1	3	4	2
	Teddy	1	4	4	0	1	0
	Totals	1	18	45	66	61	29

Average range is 3.

Average peak is 3.3.

*Sample size is 5. Each pupil did 10 sample sets.

Probability Unit

Hancock School
Dec. 15, 1966

THUMBTACK EXPERIMENT

Frequency Table for the Number of Ups for Each Team*

	Number of Ups															
	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Team 1	0	0	0	0	0	0	1	4	1	3	0	1	0	0	0	0
Team 2	0	0	0	0	0	1	0	2	1	1	1	3	1	0	0	0
Team 3	0	0	0	1	0	2	0	1	0	3	1	2	0	0	0	0
Team 4	0	0	1	0	1	1	1	0	2	3	0	0	1	0	0	0
Team 5	0	0	0	1	0	1	1	0	5	1	0	0	1	0	0	0
Team 6	0	0	0	0	0	4	2	1	1	2	0	0	0	0	0	0
Totals	0	0	1	2	1	7	5	10	10	13	2	6	3	0	0	0

Average range is 7.1666...

Average peak is 12.75

Note: Teams 1 and 4 collected data for an imaginary fourth person.

Frequency Table for the Number of Ups for Each Super - Team**

	Number of Ups															
	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Teams 1 & 2	0	0	0	0	0	0	0	2	1	1	0	2	2	0	1	1
Teams 3 & 4	0	0	1	0	2	0	0	0	1	2	0	2	0	2	0	0
Teams 5 & 6	1	0	0	1	1	1	1	1	1	1	0	0	0	2	0	0
Totals	1	0	1	1	3	1	1	3	3	4	0	4	2	4	1	1

Average range is 10.666...

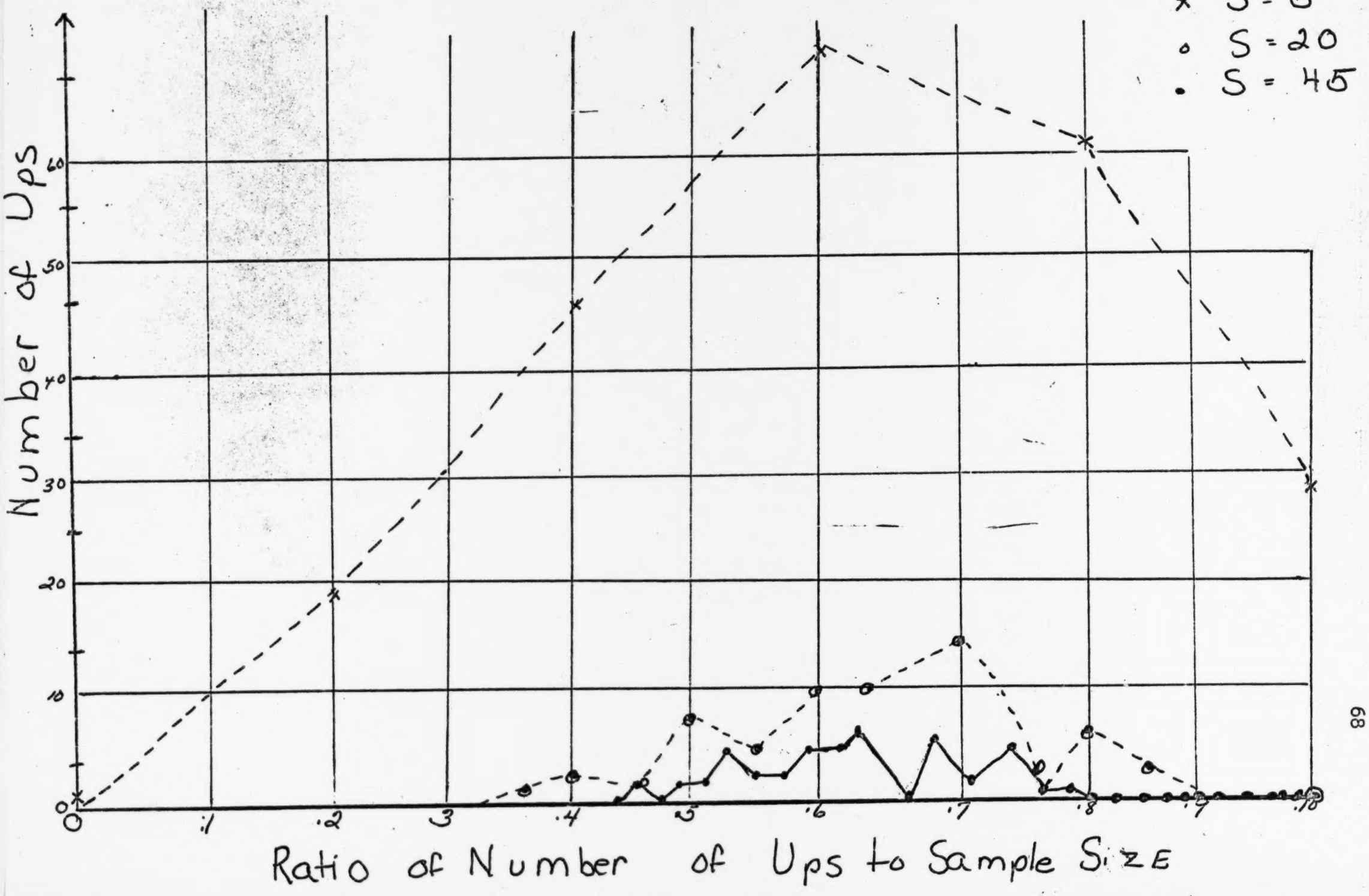
Average peak is 30.75.

Note: Each Super - Team collected data for an imaginary ninth person.

*Sample size is 20. Each Team had data for 10 sample sets.

**Sample size is 45. Each Super - Team had data for 10 sample sets.

Frequency Graph for Thumbtack Experiment



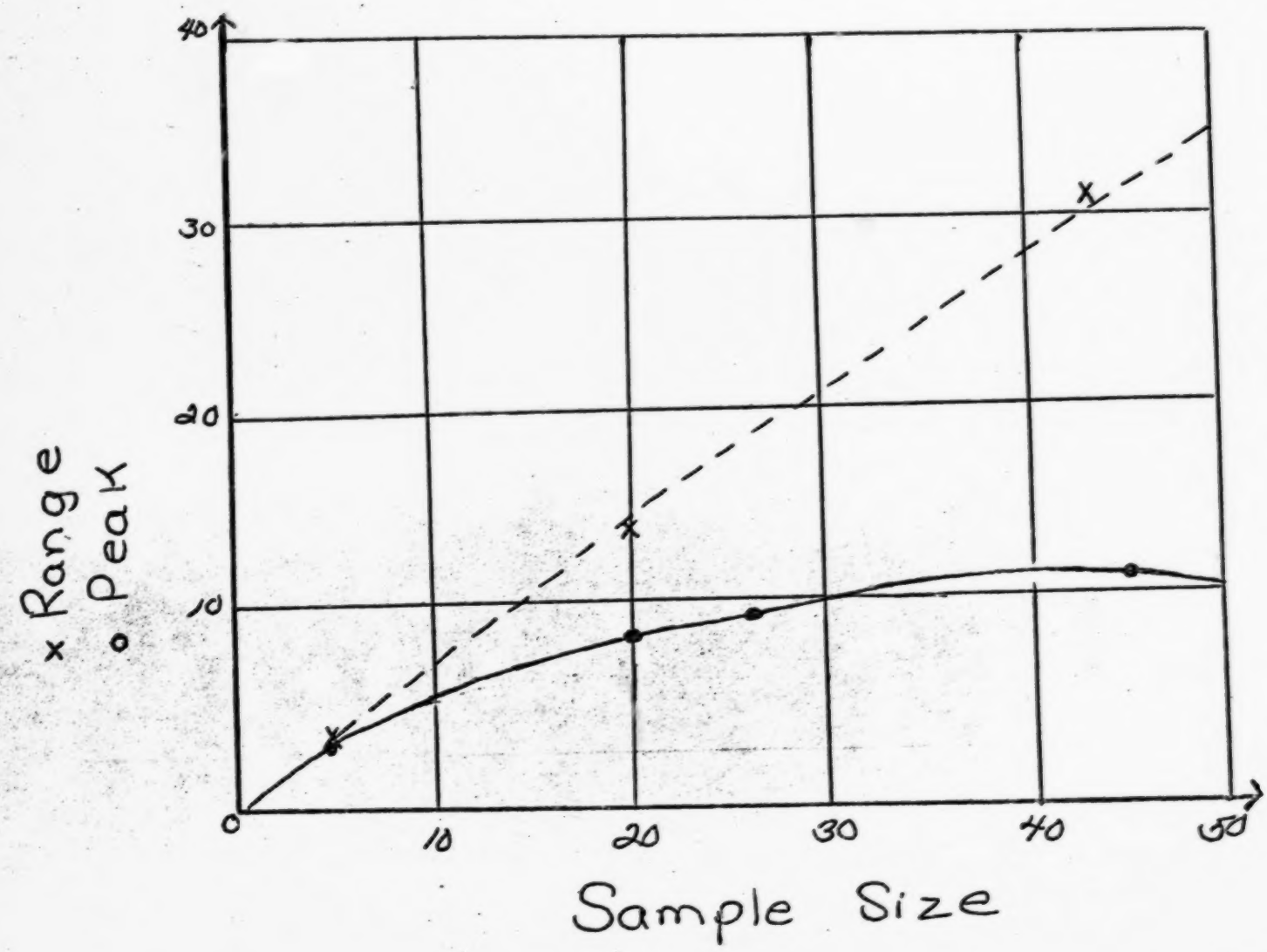
Probability Unit

Hancock School
Jan. 17, 1967

THUMBTACK EXPERIMENT

Sample Size	Average Range
5	3
20	$7.18\overline{6}$
45	$10.6\overline{6}$

Sample Size	Average Peak
5	3.3
20	12.75
45	30.75



Probability Unit

Hancock School
May, 1967

Data Presented in Class Reports

1. Two thumbtacks - 100 tosses.
   P(U) = .7 P(S) = .3
   Kim's data:

U,U	U,S	S,U	S,S
33	23	20	24

2. Three coins - 100 tosses
   P(U) = $\frac{1}{2}$ P(D) = $\frac{1}{2}$
   David R.'s data:

3 H	2 H, 1 T	1 H, 2 T	3 T
12	43	33	12

3. One coin, one die - 150 tosses
   P(1) = 1/6, P(2) = 1/6, ... , P(6) = 1/6 P(U) = 1/2, P(D) = 1/2
   Cathy's data:

	1	2	3	4	5	6
H	12	12	10	7	11	13
T	13	14	12	15	10	12

Sue's data:

	1	2	3	4	5	6
H	10	14	11	20	11	16
T	12	4	12	12	15	12

Probability Unit

Hancock School
May, 1967

Data presented in Class Reports

Helena's data:

	1	2	3	4	5	6
H	12	13	10	10	17	12
T	8	13	15	10	15	15

Sum of data in above three tables.

	1	2	3	4	5	6
H	34	39	31	37	39	41
T	33	31	39	37	40	39

4. Two dice - 400 tosses
   $P(1) = 1/6$, $P(2) = 1/6$, ... , $P(6) = 1/6$

   Teddy's Data:

Second Die

First Die	1	2	3	4	5	6
1	9	5	8	12	9	6
2	9	11	15	13	13	11
3	9	9	3	11	13	12
4	7	11	11	11	11	10
5	8	9	10	21	19	6
6	19	10	18	16	19	13

Probability Unit

Hancock School
May, 1967

Data Presented in Class Reports

Roger's Data:

First Die \ Second Die	1	2	3	4	5	6
1	10	16	9	12	11	12
2	8	6	7	14	7	8
3	12	8	11	5	8	6
4	13	10	13	16	10	7
5	10	10	11	7	17	14
6	13	10	12	16	13	13

Betsy's data:

First Die \ Second Die	1	2	3	4	5	6
1	19	10	9	10	8	13
2	15	9	17	16	8	7
3	9	11	10	6	11	7
4	8	9	10	8	7	12
5	15	12	9	10	13	11
6	10	7	11	8	6	14

Probability Unit

Hancock School
May, 1967

Data Presented in Class Reports

Karen and Alison's data: (only 252 tosses)

Second Die

	1	2	3	4	5	6
1	13	5	6	5	6	6
2	6	5	9	11	9	6
3	6	5	9	8	6	5
4	8	12	9	8	2	4
5	4	1	11	12	11	6
6	5	6	4	7	6	10

Sum of data in above four tables:

Second Die

	1	2	3	4	5	6
1	51	36	32	39	34	37
2	38	31	48	54	37	32
3	36	33	33	30	38	30
4	36	42	43	43	30	33
5	37	32	41	50	60	37
6	47	33	45	47	44	50

5. Two coins - 100 tosses
   P(H) = 1/2 P(T) = 1/2

H,H	H,T	T,H	T,T
27	23	23	27